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Supertube dynamics in diverse backgrounds

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Abstract: We study the homogeneous and time dependent dynamics of the supertube in diverse backgrounds. After deriving a general form of the hamiltonian in general background, we use a particular gauge fixing, relevant to our analysis to derive a simpler hamiltonian. We then study the homogeneous solutions of the equations of motion in various backgrounds and study the effective potential in detail.

Keywords: D-branes.

Contents

1. Introduction

The formulation of string theory in various time-dependent and cosmological backgrounds remains as one of the most important and exciting open problems for theoretical physicists. In doing so, one expects a better understanding of some of the outstanding problems and confusions of quantum gravity and cosmology. In an attempt along this direction, Sen [1] proposed a rolling tachyon solution — the decay of the unstable D-brane (brane-anti brane pair) when the tachyon rolls in the valley into the bottom of its potential (vacuum without D-branes). It has also been argued in [2] that the final state of such a decay process leads to a classical matter state, with the equation of motion of a non-interacting and pressureless dust known as 'tachyon mater' with no obvious open string excitations. These solutions, in general, are constructed by perturbing the boundary conformal field theory that describes the D-brane by an exact marginal deformation. The real time tachyon dynamics shows that the effective field theory for Dirac-Born-Infeld type captures surprisingly well many aspects of rolling tachyon solutions of full string theory. (See [3] and references therein for a detailed review of the tachyon dynamics)

Recently, Kutasov [4, 5] gave a 'geometric realization' of the open string tachyon, in the form of the rolling D-brane — the time dependent dynamics of the D-brane in the throat of NS5-branes. It has been shown that the decay process resembles astonishingly, that of the decay of unstable D-branes, when we restricts ourselves to the case when the distance between the D-brane and NS5-brane is of the order of the string length (l_s). One wonders whether the above identification is an artifact of low energy effective field theory or it can be extended to the full string theory. This was analyzed in [6] in the form of the boundary states of the in falling D-branes into the throat region of a stack of NS5-branes (the CHS geometry). The electric deformation of the process has also been realized in terms of the decay of the electrified D-brane into the NS5 throat [7, 8]. This has further been extended in [9, 10] to the case of Dp-brane background. Later on in [11, 12], this ideas were extended to the case of the Non-BPS probe branes and a careful analysis revealed the existence of a symmetry (possibly broken explicitly at the Lagrangian level [13, 14]). Hence a natural guess will be that the dynamics of the D-branes in the curved backgrounds has a much more richer structure in the geometry and in the dynamics [15], and hence likely to add some input in our understanding of string theory in time dependent curved backgrounds. See ([16] – [24]) for related studies of the D-brane dynamics and cosmological applications in various backgrounds.

In another context, the supertube [25] — the bunch of straight strings with D0-brane blown up into a supersymmetric tubular D2-brane with electric and magnetic field — has been instrumental in understanding black hole physics. For example, the quantum states of a supertube counted from the direct quantization of the Born-Infeld action near the geometrically allowed microstates with some fixed charges are shown to be in one to one correspondence with some black holes. The stability of this bound state is achieved by the non-zero angular momentum generated by the Born-Infeld electric and magnetic charges. Recently the stability of the supertubes in other curved backgrounds has also been studied in [26]. Hence in the recent surge of studying the D-brane dynamics in various backgrounds, in connection with finding out the time dependent solutions of string theory and tachyon dynamics, it seems very interesting to study the dynamics of the supertube in diverse backgrounds. But looking at the rather complicated analysis of the problem in the usual effective theory approach, we would like to take advantage of the hamiltonian formulation. This approach has been very instructive in investigating the D-branes in the strong coupling limit, by formally taking the zero tension limit [27, 28], for example. By doing this we achieve a formal hamiltonian for the D-brane motion in curved background, and the beauty of this formalism makes us comfortable to use the constraints of equations of motion in appropriate ways. We try to be as general as possible in the beginning, but while studying the motion of the tube in the background of various macroscopic objects, we use some properties of the supergravity backgrounds relevant for studying string theory, to make the analysis simpler.

The layout of the paper is as follows. In section 2, we start by deriving the Dirac-Born-Infeld action using the hamiltonian formulation. The rather complicated action can be made simpler by making use of an appropriate gauge fixing. After deriving an action for the dynamics of the tube in rather general curved backgrounds, in section 3, we focus our attention to the case of Dp-branes, NS5-branes and the fundamental string backgrounds. We study in detail the effective potential and the motion of the tube in the vicinity of these backgrounds generated by the macroscopic objects. We present our conclusions in section 3.

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2. Hamiltonian dynamics for Dp-brane in general background and its gauge fixed version

This section is devoted to the formulation of the hamiltonian formalism for Dp-brane in general curved background. We will mainly follow the very nice analysis presented in [27, 28]. The Dirac-Born-Infeld action for a Dp-brane in a general bosonic background is given by the following usual form[1]

$$S_p = -\tau_p \int d^{p+1}\xi e^{-\Phi}\sqrt{-\det \mathbf{A}}\,, \tag{2.1}$$

where

$$\mathbf{A}_{\mu\nu} = \gamma_{\mu\nu} + F_{\mu\nu}, \qquad \mu,\nu = 0,\dots,p\,, \tag{2.2}$$

and where τ_p is Dp-brane tension. The induced metric $\gamma_{\mu\nu}$ and the induced field strengths on the worldvolume $F_{\mu\nu}$ are given by the following expressions

$$\begin{aligned}\gamma_{\mu\nu} &= \partial_\mu X^M \partial_\nu X^N g_{MN}\,,\\ F_{\mu\nu} &= \partial_\mu X^M \partial_\nu X^N b_{MN} + \partial_\mu A_\nu - \partial_\nu A_\mu\,.\end{aligned} \tag{2.3}$$

Where $g_{MN}(X(\xi))\,, b_{MN}(X(\xi))\,, M,N = 0,\dots,9$ are metric and NS-NS two form field that are in general functions of the embedding coordinates $X^M(\xi)$. Let us now calculate the conjugate momenta from (2.1)

$$\begin{aligned}P_M(\xi) &= \frac{\delta S}{\delta\partial_0 X^M(\xi)} = -\frac{e^{-\Phi}}{2}\sqrt{-\det\mathbf{A}}\partial_\nu X^N\left(g_{MN}(\mathbf{A}^{-1})^{(\nu 0)} + b_{MN}(\mathbf{A}^{-1})^{[\nu 0]}\right),\\ \pi^a(\xi) &= \frac{\delta S}{\delta\partial_0 A_a(\xi)} = -\frac{e^{-\Phi}}{2}\sqrt{-\det\mathbf{A}}(\mathbf{A}^{-1})^{[a0]}\,, \qquad a,b = 1,\dots,p\,,\\ \pi^0(\xi) &= \frac{\delta S}{\delta\partial_0 A_0(\xi)} = -\frac{e^{-\Phi}}{2}\left((\mathbf{A}^{-1})^{00} - (\mathbf{A}^{-1})^{00}\right)\sqrt{-\det\mathbf{A}} = 0\,,\end{aligned} \tag{2.4}$$

where we have introduced the symmetric and antisymmetric form of any (p+1)×(p+1) matrix A as

$$\mathrm{A}^{(\mu\nu)} = \mathrm{A}^{\mu\nu} + \mathrm{A}^{\nu\mu}\,, \qquad \mathrm{A}^{[\mu\nu]} = \mathrm{A}^{\mu\nu} - \mathrm{A}^{\nu\mu}\,. \tag{2.5}$$

For our purpose it is useful to define the conjugate momenta Π_M as

$$\Pi_M = P_M + P^a B_{aM} = -\frac{e^{-\Phi}}{2}V\sqrt{-\det\mathbf{A}}\gamma_{M\nu}(\mathbf{A}^{-1})^{(\nu 0)}\,, \tag{2.6}$$

where

$$\gamma_{M\nu} = g_{MN}\partial X^N_\nu\,, B_{aM} = \partial_a X^N B_{NM}\,. \tag{2.7}$$

Using these expressions it is easy to see that following constraints are obeyed

$$\begin{aligned}\Pi_M\partial_a X^M + \pi^a F_{ab} &= 0\,,\\ \Pi_M g^{MN}\Pi_N + \pi^a\gamma_{ab}\pi^b + e^{-2\Phi}\tau_p^2\det\mathbf{A}_{ab} &= 0\,,\\ \pi^0 &= 0\,.\end{aligned} \tag{2.8}$$

[1] We do not consider the Wess-Zummino term since in the examples studied below the coupling to Ramond-Ramond fields is not important.

The corresponding hamiltonian for Dp-brane in curved background takes the following form

$$H = \int d^p \xi \mathcal{H}(\xi) , \tag{2.9}$$

where the hamiltonian density ($\mathcal{H}$) is given by

$$\mathcal{H} = \pi^i \partial_i A_0 + \sigma \pi^0 + \rho^a (\Pi_M \partial_a X^M + F_{ab} \pi^b) + \lambda (\Pi_M g^{MN} \Pi_N + \pi^a \gamma_{ab} \pi^b + e^{-2\Phi} \tau_p^2 \det \mathbf{A}_{ab}) , \tag{2.10}$$

where σ, ρ^a and λ are Lagrange multipliers for the constraints. More precisely, the final hamiltonian is just the sum of constraints, in agreement with the diffeomorphism invariance of the original lagrangian.

2.1 Gauge fixing problem

Since the original DBI action is diffeomorphism invariant, it is convenient to use this symmetry to reduce the number of independent equations of motions. This procedure is commonly known as gauge fixing.

To proceed we can without loss of generality presume that the metric takes diagonal form. Then we consider the equation of motion for X^M

$$\partial_0 X^M = \frac{\delta H}{\delta P_M(\xi)} = \rho^a \partial_a X^M + 2\lambda g^{MN} \Pi_N . \tag{2.11}$$

Usually the static gauge is imposed at the level of the action and lagrangian. In our case, however, we would like to perform the similar procedure at the level of canonical equations of motion. Since in the lagrangian formalism the static gauge is imposed by the constraints $X^\mu = \xi^\mu \, , \mu = 0, \dots, p$, it is natural to solve the canonical equations of motion for X^μ by the ansatz

$$\partial_0 X^0 = 1 \, , \partial_a X^b = \delta_a^b \, , a = 1, \dots, p . \tag{2.12}$$

Then the equations of motion for X^μ simplify as

$$\Pi_0 = \frac{g_{00}}{2\lambda} , \qquad \Pi_a = -\frac{\rho^a g_{aa}}{2\lambda} \qquad \text{(No summation over a)} . \tag{2.13}$$

Using these results we get

$$\Pi_M \partial_a X^M = -\frac{\rho^b g_{ba}}{2\lambda} + \Pi_s \partial_a X^s \tag{2.14}$$

where $s, r, t \dots$ label the directions transverse to Dp-brane. If we ignore the terms in the hamiltonian that enforce Gauss law constraints and the vanishing of π^0 we obtain [2]

$$\mathcal{H} = \frac{g_{00}}{4\lambda} - \frac{\rho^a g_{ab} \rho^b}{4\lambda} + \rho^a b_a + \lambda D , \tag{2.16}$$

[2]To see this, note that the equation of motion for A_0 leads to the "Gauss law" constraints

$$\frac{\delta H}{\delta A_0(\mathbf{x})} = 0 \Rightarrow \partial_a \pi^a = 0 \tag{2.15}$$

and variation of the hamiltonian with respect to σ implies $\pi^0 = 0$.

where

$$
\begin{aligned}
b_a &= \Pi_s \partial_a X^s + F_{ab}\pi^b \,, \\
D &= \Pi_r g^{rs}\Pi_s + \pi^a \gamma_{ab}\pi^b + e^{-2\Phi}\tau_p^2 \det \mathbf{A}_{ab} \,.
\end{aligned} \tag{2.17}
$$

In order to enforce the variation with the Lagrange multipliers we should consider the Dp-brane action

$$
S_p = -\int d^{p+1}\xi \, \mathcal{L} \tag{2.18}
$$

where we express $\mathcal{L}$ as a Lagrange transformation of the hamiltonian density (2.16) so that we obtain the action in the form

$$
S_p = -\int d^{p+1}\xi \left(P_M \partial_0 X^M + \pi_a \partial_0 A^a - \mathcal{H} \right) \tag{2.19}
$$

Now we replace P_μ with the help of Π_μ given above. Using the fact that $\partial_0 X^\mu = \delta_0^\mu$ we obtain an action for P_s, X^s that contains the Lagrange multipliers λ, ρ^a

$$
S_p = -\int d^{p+1}\xi \left(P_s \partial_0 X^s + \pi^a \partial_0 A_a - \pi^a B_{a0} + \frac{g_{00}}{4\lambda} + \frac{\rho^a g_{ab}\rho^b}{4\lambda} - \lambda D - \rho^a b_a \right) . \tag{2.20}
$$

Now the variation with respect to ρ^a implies (remember that g_{ab} is diagonal)

$$
\rho^a = 2\lambda g^{ab} b_b \,. \tag{2.21}
$$

If we then insert (2.21) back into the action we get

$$
S = -\int d^{p+1}\xi \left(P_s \partial_0 X^s + \pi^a \partial_0 A_a - \pi^a B_{a0} + \frac{g_{00}}{4\lambda} - \lambda (D + b_a g^{ab} b_b) \right) . \tag{2.22}
$$

Now the variation with respect to λ implies

$$
\lambda = \frac{1}{2}\sqrt{\frac{-g_{00}}{b_a g^{ab} b_b + D}} \,. \tag{2.23}
$$

Inserting back to the action we get finally

$$
S = -\int d^{p+1}\xi \left(P_s \partial_0 X^s + \pi^a \partial_0 A_a - \sqrt{-g_{00}}\sqrt{b_a g^{ab} b_b + D} - \pi^a B_{a0} \right) \tag{2.24}
$$

from which we obtain the hamiltonian density for transverse variables Y^s, P_s and gauge field p^a, A_a in the form

$$
\begin{aligned}
\mathcal{H} &= \sqrt{-g_{00}}\sqrt{\mathcal{K}} + \Pi^a B_{a0} \,, \\
\mathcal{K} &= \Pi_r g^{rs}\Pi_s + \pi^a \gamma_{ab}\pi^b + b_a g^{ab} b_b + e^{-2\Phi}\tau_p^2 \det \mathbf{A}_{ab} \,, \\
\Pi_s &= P_s + \pi^a B_{as} \,, b_a = \Pi_s \partial_a X^s + F_{ab}\pi^b \,.
\end{aligned} \tag{2.25}
$$

Now we are ready, using the hamiltonian density (2.25) to study the dynamics of the supertube in the various supergravity background.

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3. Supertube in diverse backgrounds

By standard treatment, the supertube in flat spacetime is the D2-brane that is stretched in one particular direction, say z direction and that have arbitrary shape in the transverse R^8 space. In the case when we study the supertube dynamics in the background of some macroscopic objects (Dp-branes, NS5-branes or fundamental strings) the situation is slightly more complicated. On the other hand since these backgrounds have generally manifest rotational invariance of the transverse R^{9-k} space, where k is the spatial dimension of given objects, it is natural to simplify the analysis by presuming that the supertube has its base in the (x^8, x^9)-plane.[3] In this plane we introduce the polar coordinates

$$x^8 = R\cos\phi\,, \qquad x^9 = R\sin\phi\,. \tag{3.1}$$

Then the embedding coordinates are $R(\sigma,t)\,, X^m\,, m = p+1,\dots,7$, that parameterize the position of D2-brane in the directions transverse to given macroscopic objects of spatial dimension p which are however transverse to the (x^8, x^9) plane. We also introduce the coordinates $Y^u\,, (u,v = 2,\dots,p)$, where Y^u parameterize the position of supertube in the space parallel with Dp-brane. Since the B field is also zero, the hamiltonian for such a Dp-brane takes the form

$$H = \int dtdzd\sigma\mathcal{H} = \int dtdzd\sigma\left[\sqrt{-g_{00}}\sqrt{\mathcal{K}}\right], \tag{3.2}$$

with

$$\begin{aligned}\mathcal{K} = \ &\pi_a g^{ab}\pi_b + p_R g^{RR} p_R + p_m g^{mn} p_n + p_u g^{uv} p_v + b_a g^{ab} b_b + \\ &+(\pi^a\partial_a R)g_{RR}(\pi^b\partial_b R) + (\pi^a\partial_a Y^u)g_{uv}(\pi^b\partial_b Y^v) + \\ &+(\pi^a\partial_a X^m)g_{mn}(\pi^b\partial_b X^n) + e^{-2\Phi}\tau_2^2\det\mathbf{A}_{ab}\,,\end{aligned} \tag{3.3}$$

and

$$\mathbf{A}_{ab} = g_{ab} + g_{RR}\partial_a R\partial_b R + g_{mn}\partial_a X^m\partial_b X^n + g_{uv}\partial_a Y^u\partial_b Y^v + F_{ab}\,, \qquad a,b = z,\phi \tag{3.4}$$

with

$$b_a = F_{ab}\pi^b + \partial_a R\, p_R + \partial_a X^m p_m + \partial_a Y^u p_u\,. \tag{3.5}$$

Using the hamiltonian (3.2) the equation of motions for A_a and π^a take the form

$$\begin{aligned}\dot{A}_a(\mathbf{x}) &= \frac{\delta H}{\delta\pi^a(\mathbf{x})} \\ &= \sqrt{-g_{00}}\frac{g_{ab}\pi^b + \partial_a X^m g_{mn}(\pi^b\partial_b X^n) + \partial_a Y^u g_{uv}(\pi^b\partial_b Y^v) + F_{ab}g^{bc}b_c}{\sqrt{\mathcal{K}}}\,,\end{aligned} \tag{3.6}$$

[3]We restrict ourselves to the case of background Dp-branes with $p < 5$ so that the coupling of the D2-brane to the background Ramond-Ramond field vanishes.

$$\begin{aligned}\dot{\pi}^a(\mathbf{x}) &= -\frac{\delta H}{\delta A_a(\mathbf{x})} = \partial_c\left[\frac{\sqrt{-g_{00}}\pi^a g^{cb}b_b}{\sqrt{\mathcal{K}}}\right] - \partial_c\left[\frac{\sqrt{-g_{00}}\pi^c g^{ab}b_b}{\sqrt{\mathcal{K}}}\right] + \\ &+\frac{1}{2}\partial_c\left[\frac{e^{-2\Phi}\tau_2^2\sqrt{-g_{00}}}{\sqrt{\mathcal{K}}}(\mathbf{A}^{-1})^{ac}\det\mathbf{A}_{ab}\right] - \\ &-\frac{1}{2}\partial_c\left[\frac{e^{-2\Phi}\tau_2^2\sqrt{-g_{00}}}{\sqrt{\mathcal{K}}}(\mathbf{A}^{-1})^{ca}\det\mathbf{A}_{ab}\right]. \end{aligned} \tag{3.7}$$

Further, the equations of motion for the embedding coordinates are

$$\begin{aligned}\dot{R}(\mathbf{x}) &= \frac{\delta H}{\delta p_R(\mathbf{x})} = \frac{\sqrt{-g_{00}}}{\sqrt{\mathcal{K}}}\left(g^{RR}p_R + \partial_i R g^{ab}b_b\right), \\ \dot{X}^m(\mathbf{x}) &= \frac{\delta H}{\delta p_m(\mathbf{x})} = \frac{\sqrt{-g_{00}}}{\sqrt{\mathcal{K}}}\left(g^{mn}p_n + \partial_a X^m g^{ab}b_b\right), \\ \dot{Y}^u(\mathbf{x}) &= \frac{\delta H}{\delta p_u(\mathbf{x})} = \frac{\sqrt{-g_{00}}}{\sqrt{\mathcal{K}}}\left(g^{uv}p_v + \partial_a X^u g^{ab}b_b\right), \end{aligned} \tag{3.8}$$

while the equation of motion for p_m , p_u and p_R are

$$\begin{aligned}\dot{p}_m(\mathbf{x}) = -\frac{\delta H}{\delta X^m(\mathbf{x})} = &\partial_a\left[\frac{\sqrt{-g_{00}}}{\sqrt{\mathcal{K}}}\pi^a g_{mn}(\pi^b\partial_b X^n)\right] + \\ &+\partial_a\left[\frac{\sqrt{-g_{00}}}{\sqrt{\mathcal{K}}}e^{-2\Phi}\tau_2^2 g_{mn}\partial_b X^n(\mathbf{A}^{-1})^{ba}\det\mathbf{A}_{ab}\right] + \frac{\delta g_{00}}{\delta X^m}\frac{\sqrt{\mathcal{K}}}{2\sqrt{-g_{00}}} - \\ &-\frac{\sqrt{-g_{00}}}{2\sqrt{\mathcal{K}}}\Big(\pi^a\frac{\delta g_{ab}}{\delta X^m}\pi^b + p_R\frac{\delta g^{RR}}{\delta X^m}p_R + p_n\frac{\delta g^{no}}{\delta X^m}p_o + \\ &\qquad + p_u\frac{\delta g^{uv}}{\delta X^m}p_v + \frac{\delta[e^{-2\Phi}]}{\delta R}\tau_2^2\det\mathbf{A}_{ab} + \\ &\qquad + e^{-2\Phi}\tau_2^2\left[\frac{\delta g_{RR}}{\delta X^m}(\partial_a R\partial_b R) + \frac{\delta g_{no}}{\delta X^m}(\partial_a X^n\partial_b X^o) + \right. \\ &\qquad \left. + \frac{\delta g_{uv}}{\delta X^m}(\partial_a Y^u\partial_b Y^v)\right](\mathbf{A}^{-1})^{ba}\det\mathbf{A}_{ab}\Big), \end{aligned} \tag{3.9}$$

$$\begin{aligned}\dot{p}_u(\mathbf{x}) &= -\frac{\delta H}{\delta Y^u(\mathbf{x})} \\ &= \partial_a\left[\frac{\sqrt{-g_{00}}}{\sqrt{\mathcal{K}}}\pi^a g_{uv}(\pi^b\partial_b Y^v)\right] + \partial_a\left[\frac{\sqrt{-g_{00}}}{\sqrt{\mathcal{K}}}e^{-2\Phi}\tau_2^2 g_{uv}\partial_b Y^v(\mathbf{A}^{-1})^{ba}\det\mathbf{A}_{ab}\right] \end{aligned} \tag{3.10}$$

and

$$\begin{aligned}\dot{p}_R(\mathbf{x}) = -\frac{\delta H}{\delta R(\mathbf{x})} = &\partial_a\left[\frac{\sqrt{-g_{00}}}{\sqrt{\mathcal{K}}}\pi^a g_{RR}(\pi^b\partial_b R)\right] + \\ &+\partial_a\left[\frac{\sqrt{-g_{00}}}{\sqrt{\mathcal{K}}}e^{-2\Phi}\tau_2^2 g_{RR}\partial_b R(\mathbf{A}^{-1})^{ba}\det\mathbf{A}_{ab}\right] + \frac{\delta g_{00}}{\delta R}\frac{\sqrt{\mathcal{K}}}{2\sqrt{-g_{00}}} - \\ &-\frac{\sqrt{-g_{00}}}{2\sqrt{\mathcal{K}}}\Big(\pi^a\frac{\delta g_{ab}}{\delta R}\pi^b + p_R\frac{\delta g^{RR}}{\delta R}p_R + \\ &\qquad + p_m\frac{\delta g^{mn}}{\delta R}p_n + p_u\frac{\delta g^{uv}}{\delta R}p_v + \frac{\delta[e^{-2\Phi}]}{\delta R}\tau_2^2\det\mathbf{A}_{ab} + \end{aligned}$$

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$$+ e^{-2\Phi}\tau_2^2\left[\frac{\delta g_{RR}}{\delta R}(\partial_a R\partial_b R)+\frac{\delta g_{mn}}{\delta R}(\partial_a X^m\partial_b X^n)+\right.$$
$$\left.+\frac{\delta g_{uv}}{\delta R}(\partial_a Y^u\partial_b Y^v)\right](\mathbf{A}^{-1})^{ba}\det\mathbf{A}_{ab}\Big). \quad (3.11)$$

Note that generally the metric is function of the expression $R^2 + X^m X_m$. On the other hand thanks to the symmetry of the problem with respect to z direction it is natural to consider the modes that are z independent. We also take the ansatz for the gauge field in the form

$$F = \partial_0 A_z(\phi)dt\wedge dz + B(\phi)dz\wedge d\phi\,, A_\phi = Bz\,. \quad (3.12)$$

Then the matrix $\mathbf{A}_{ab}$ takes the form

$$\mathbf{A}_{ab} = \begin{pmatrix} g_{zz} & B \\ -B & g_{\phi\phi}+g_{RR}(\partial_\phi R)^2+g_{mn}\partial_\phi X^m\partial_\phi X^n+g_{uv}\partial_\phi Y^u\partial_\phi Y^v\end{pmatrix} \quad (3.13)$$

so that

$$\det\mathbf{A}_{ab} = g_{zz}\left(g_{\phi\phi}+g_{RR}(\partial_\phi R)^2+g_{mn}\partial_\phi X^m\partial_\phi X^n+g_{uv}\partial_\phi Y^u\partial_\phi Y^v\right)+B^2 \quad (3.14)$$

and

$$(\mathbf{A}^{-1})^{ab} = \frac{1}{\det\mathbf{A}}\begin{pmatrix} g_{\phi\phi}+g_{RR}(\partial_\phi R)^2+g_{mn}\partial_\phi X^m\partial_\phi X^n+g_{uv}\partial_\phi Y^u\partial_\phi Y^v & B \\ -B & g_{zz}\end{pmatrix}. \quad (3.15)$$

With this notation the equation of motion for π^z takes the form

$$\dot{\pi}^z = \partial_\phi\left[\frac{\sqrt{-g_{00}}}{\sqrt{\mathcal{K}}}g^{\phi\phi}(-B\pi^z+\partial_\phi X^m p_m+\partial_\phi Y^u p_u)\right]+\partial_\phi\left[e^{-2\Phi}\tau_2^2\frac{\sqrt{-g_{00}}B}{\sqrt{\mathcal{K}}}\right]. \quad (3.16)$$

As it is clear from the equation above π^z is generally time dependent. On the other hand we can show that the quantity $n=\oint d\phi\pi^z$ is conserved since

$$\dot{n} = \oint d\phi\dot{\pi}^z = \oint d\phi\partial_\phi[\ldots] = 0\,. \quad (3.17)$$

The similar case occurs for p_u where $\dot{p}_u(\mathbf{x})\neq 0$

$$\dot{p}_u = \partial_\phi\left[\frac{\sqrt{-g_{00}}}{\sqrt{\mathcal{K}}}e^{-2\Phi}\tau_2^2 g_{uv}(\partial_\phi Y^v)g_{zz}\right] \quad (3.18)$$

while the the total momentum P_u is conserved

$$P_u \equiv \oint d\phi p_u \Rightarrow \dot{P}_u = \oint d\phi\dot{p}_u = \oint d\phi\partial_\phi[(\ldots)] = 0\,. \quad (3.19)$$

To simplify further the analysis of the time dependent evolution of supertube we will restrict ourselves to the case of homogenous fields on the worldvolume of D2-brane:

$$\partial_\phi R = \partial_\phi X^m = \partial_\phi Y^u = \partial_\phi\pi = 0\,. \quad (3.20)$$

As it is now clear from (3.18) and (3.16) $\pi^z \equiv \Pi$ and p_u are conserved. On the other hand the equation of motion for $A_z\,, A_\phi$ are equal to

$$\begin{aligned}\dot{A}_z &= E = \frac{\sqrt{-g_{00}}}{\sqrt{\mathcal{K}}}(g_{zz} + g^{\phi\phi}B^2)\Pi^2\,,\\ \dot{A}_\phi &= 0\,,\end{aligned} \tag{3.21}$$

where the second equation above implies that $B = \partial_z A_\phi$ is conserved as well. Then we get simple form of the hamiltonian density $\mathcal{H}$

$$\mathcal{H} = \sqrt{-g_{00}}\sqrt{p_R g^{RR} p_R + p_m g^{mn} p_n + p_u g^{uv} p_v + \Pi g_{zz}\Pi + b_\phi g^{\phi\phi} b_\phi + e^{-2\Phi}\tau_2^2\,(g_{zz}g_{\phi\phi} + B^2)}, \tag{3.22}$$

where

$$b_\phi = F_{\phi z}\pi^z = -B\Pi\,. \tag{3.23}$$

For supertube in Dp-brane background we have

$$\begin{aligned}g_{00} &= -H_p^{-1/2}\,, g_{zz} = H_p^{-1/2}\,, g_{\phi\phi} = H_p^{1/2}R^2\,,\\ g_{uv} &= H_p^{-1/2}\delta_{uv}\,, g_{mn} = H_p^{1/2}\delta_{mn}\,, g_{RR} = H_p^{1/2}\,,\\ e^{-2\Phi} &= H_p^{\frac{p-3}{2}}\end{aligned} \tag{3.24}$$

where the harmonic function for N Dp-branes is given by[4]

$$H_p = 1 + \frac{N}{(R^2 + X^m X_m)^{(7-p)/2}} \tag{3.25}$$

and hence the hamiltonian density takes the form

$$\mathcal{H} = \sqrt{\frac{\Pi^2}{H_p}\left(1 + \frac{B^2}{R^2}\right) + \frac{p_R^2}{H_p} + \frac{p_m p^m}{H_p} + p_u p^u + \tau_2^2 H_p^{\frac{p-4}{2}}\,[R^2 + B^2]} \tag{3.26}$$

that in the static case ($p_u = p_R = p_m = 0$) reduces to the hamiltonian studied in [26]. For letter purposes it is also useful to define conserved energy $\mathcal{E}$ through the relation

$$\mathcal{E} = \oint \mathcal{H} = 2\pi\mathcal{H} \Rightarrow \mathcal{H} = \frac{\mathcal{E}}{2\pi}\,. \tag{3.27}$$

As it is clear from the form of the harmonic function H_p the background has SO$(7-p)$ symmetry in the transverse subspace labelled with coordinates X^m. Then it is natural to simplify the analysis by restricting the dynamics of the supertube to the (x^6, x^7) plane and introduce second polar coordinates as

$$x^6 = \rho\cos\psi\,, x^7 = \rho\sin\psi \tag{3.28}$$

with corresponding metric components

$$g_{\rho\rho} = H_p^{1/2}\,, g_{\psi\psi} = H_p^{1/2}\rho^2\,. \tag{3.29}$$

[4]We work in units $l_s = 1$.

Then the hamiltonian density $\mathcal{H}$ can be written as

$$\mathcal{H} = \sqrt{\frac{\Pi^2}{H_p}\left(1+\frac{B^2}{R^2}\right) + \frac{p_R^2}{H_p} + \frac{p_\rho^2}{H_p} + \frac{p_\psi^2}{H_p\rho^2} + p_u p^u + \tau_2^2 H_p^{\frac{p-4}{2}}\,[R^2+B^2]}. \tag{3.30}$$

Since the hamiltonian does not explicitly depend on ψ we get immediately that p_ψ is conserved:

$$\dot{p}_\psi = -\frac{\delta H}{\delta \psi} = 0\,. \tag{3.31}$$

Then the equations of motion for ρ, R take the form

$$\dot{R} = \frac{\delta H}{\delta p_R} = \frac{2\pi p_R}{H_p \mathcal{E}}\,,$$
$$\dot{\rho} = \frac{\delta H}{\delta p_\rho} = \frac{2\pi p_\rho}{H_p \mathcal{E}} \tag{3.32}$$

On the other hand the equation of motion for p_R, p_ρ are much more complicated thanks to the nontrivial dependence of H on ρ, R. In fact it is very complicated to solve these equations in the full generality. Let us rather consider the special case when we will study the dynamics of R only. In order to do this we should find the stable values for ρ. Let us then consider the equation of motion for p_ρ

$$\dot{p}_\rho = -\frac{1}{2\sqrt{\mathcal{K}}}\frac{\delta \mathcal{K}}{\delta H_p}\frac{\delta H_p}{\delta \rho} + \frac{p_\psi^2}{\rho^3 H_p\sqrt{\mathcal{K}}} = -\frac{1}{\sqrt{\mathcal{K}}}\left[\frac{N(p-7)\rho}{(R^2+\rho^2)^{(8-p)/2}}\frac{\delta \mathcal{K}}{\delta H_p} - \frac{p_\psi^2}{\rho^3 H_p}\right]. \tag{3.33}$$

We see that the momentum p_ρ is equal to zero for $\rho = \psi = 0$. The question is whether there exist closed orbits with $p_\psi \neq 0$ for which $p_\rho = 0$. Looking at the equation above it is clear that we find ρ as function of N, conserved momenta p_u, p_ψ and, most importanly, as a function of R. Then it follows that the resulting hamiltonian is complicated function of R. Even if it would be certainly interesting to study the properties of supertube with nonzero p_ψ we will restrict ourselved in this paper to the case of $p_\psi = \rho = 0$.

Analogously, we can consider the situation when $p_R = 0$ that corresponds some particular value of $R_{stat} = R(N, p_u, p_\psi, \rho)$ and study the time dependence of ρ. As in the previous case we leave the study of this problem for future.

Now let us consider the case when $p_\rho = p_\psi = \rho = 0$. In this case the study of the supertube dynamics reduces to the study of the time dependence of R. Then it is natural to write the hamiltonian density as as $\mathcal{H} = \sqrt{\frac{p_R^2}{H_p} + V}$ where $\sqrt{V} = \mathcal{H}(p_R = 0)$. Then we can write the equation of motion for R in the following form

$$\dot{R} = \frac{1}{\mathcal{E}\sqrt{H_p}}\sqrt{\mathcal{E}^2 - (2\pi)^2 V}\,, \frac{2}{2}\dot{R}^2 + V_{\text{eff}} = 0\,, \tag{3.34}$$

where

$$V_{\text{eff}} = \frac{1}{H_p}\left(\frac{(2\pi)^2 V}{\mathcal{E}^2} - 1\right). \tag{3.35}$$

Then in order to obtain qualitative character of the dynamics of he supertube we use the observation [29] that the equation given above corresponds to the conservation of energy for massive particle with mass ($m = 2$) with the effective potential V_{eff} with total zero energy. We will be interested in two cases corresponding to $p = 2, 4$.

3.1 D2-brane background

In this case the effective potential takes the form

$$V = (R^2 + B^2)\left(\frac{R^3}{R^5 + N}\right)(\Pi^2 + \tau_2^2 R^2) + p_u p^u \tag{3.36}$$

and hence V_{eff} is equal to

$$V_{\text{eff}} = \frac{R^5}{R^5 + N}\left(\frac{4\pi^2}{\mathcal{E}^2}(R^2 + B^2)\frac{R^3}{R^5 + N}(\Pi^2 + \tau_2^2 R^2) + \frac{4\pi^2 p_u p^u}{\mathcal{E}^2} - 1\right). \tag{3.37}$$

It is clear that the particle with zero energy can move in the interval between the points where $V_{\text{eff}} = 0$. First of all, the asymptotic behavior of V_{eff} is as follows

$$\begin{aligned} V_{\text{eff}} &\approx \frac{R^5}{N}\left(\frac{4\pi^2 B^2 \Pi^2 \tau_2 R^3}{\mathcal{E}^2} + \frac{4\pi^2 p_u p^u}{\mathcal{E}^2} - 1\right), \qquad && R \to 0\,, \\ V_{\text{eff}} &\approx \frac{4\pi^2}{\mathcal{E}^2} R^2\,, && R \to \infty\,. \end{aligned} \tag{3.38}$$

Few comments are in order. From the above limits we can see that the potential approaches 0 for $R \to 0$ from below. Then since the potential blows up for $R \to \infty$ there should exists the point R_T where the potential vanishes: $V_{\text{eff}}(R_T) = 0$. In order to study the dynamics around this point we introduce the variable r through the substitution $R = r + R_T$ and insert it to the expression for conservation of energy. Using the fact that near the turning point we have
$V_{\text{eff}}(R) = V_{\text{eff}}(R_T) + V'_{\text{eff}}(R_T)r = V'_{\text{eff}}(R_T)r$ where we have used $V_{\text{eff}}(R_T) = 0$. Then we get the equation of motion for r

$$\dot{r}^2 = -V'_{\text{eff}}(R_T)r \Rightarrow \dot{r} = \pm\sqrt{-V'_{\text{eff}}(R_T)r}\,. \tag{3.39}$$

Since $V'_{\text{eff}}(R_T) > 0$ it follows from the equation above that r should be negative. Integrating the above equation we get

$$r = -\frac{1}{4}\left(r_0 \mp \sqrt{V'_{\text{eff}}(R_T)}t\right)^2. \tag{3.40}$$

if we demand that for $t = 0$ the particle reaches its turning point we have $r_0 = 0$ and hence we obtain

$$r = -\frac{V_{\text{eff}}}{4}t^2 \tag{3.41}$$

so that for small negative t (in order to trust $r \ll 1$) the particle approaches its turning point and then it moves back.

As follows from the properties of the effective potential V_{eff} [5] there should also exist the point where the potential reaches its local minimum corresponding to $V'_{eff}(R_m) = 0$. Again, if we introduce r as $R = r + R_m$ we get

$$V_{\text{eff}}(R) = V_{\text{eff}}(R_m) + \frac{1}{2}V''_{\text{eff}}(R_m)r^2 \tag{3.42}$$

[5] Namely, since the potential reaches zero for $R \to 0$ from below and since the potential blows up for $R \to \infty$ there should certainly exists the point where $V'_{\text{eff}}(R_m) = 0$.

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where $V_{\rm eff}(R_m) \equiv A < 0$ and $\frac{1}{2}V''_{\rm eff}(R_m) \equiv B > 0$ and hence the conservation of energy implies the following differential equation

$$\frac{dr}{\sqrt{1+\frac{B}{A}r^2}} = \pm\sqrt{-A}\,. \tag{3.43}$$

The above equation has a solution for r that is

$$r = \sqrt{\frac{|A|}{B}}\sin\sqrt{B}t = \sqrt{\frac{2|V_{\rm eff}(R_m)|}{V''_{\rm eff}(R_m)}}\sin\sqrt{\frac{V''_{\rm eff}(R_m)}{2}}t\,. \tag{3.44}$$

In other words if we have a supertube inserted close to its local minimum position we observe that the supertube will fluctuate around this point with harmonic oscillations.

Finally, we will consider the case $R \to 0$. Since now the effective potential is given by

$$V_{\rm eff} = \frac{R^5}{N}\left(\frac{4\pi^2 p_u p^u}{\mathcal{E}^2} - 1\right), \tag{3.45}$$

the time evolution equation for R is

$$\dot{R}^2 = -\frac{R^5}{N}\left(\frac{4\pi^2 p_u p^u}{\mathcal{E}^2} - 1\right) \Rightarrow \frac{1}{R^3} = \left(C \mp \frac{3t}{2}\sqrt{\frac{1}{N}\left(1 - \frac{4\pi^2 p_u p^u}{\mathcal{E}^2}\right)}\right)^2, \qquad C^2 = \frac{1}{R_0^3}, \tag{3.46}$$

where R_0 is the position of supertube in time $t = 0$. However since we demand that $R \to 0$ it is clear that the above solution is valid in case of large positive or negative t and we obtain following assymptotic behaviour of R

$$R \sim \frac{1}{t^{2/3}}\frac{1}{\left[\frac{9}{4N}\left(1 - \frac{4\pi^2 p_u p^u}{\mathcal{E}^2}\right)\right]^{1/3}}. \tag{3.47}$$

This result shows that supertube approaches the worldvolume of N D2-branes for $t \to \infty$.

3.2 D4-brane background

The situation of supertube in D4-brane background is similar to the case of D2-brane background. Namely, the potential V takes the form

$$V = (R^2 + B^2)\left[\frac{\Pi^2 R}{R^3 + N} + \tau_2^2\right] + p_u p^u \tag{3.48}$$

and hence the effective potential takes the form

$$V_{\rm eff} = \frac{R^3}{(N + R^3)}\left(\frac{4\pi^2}{\mathcal{E}^2}(R^2 + B^2)\left[\frac{\Pi^2 R}{R^3 + N} + \tau_2^2\right] + \frac{4\pi^2 p_u p^u}{\mathcal{E}^2} - 1\right). \tag{3.49}$$

from which we again obtain the asymptotic behavior

$$\begin{aligned} V_{\rm eff} &\sim \frac{R^3}{N}\left(\frac{4\pi^2(B^2\tau_2^2 + p_u^2)}{\mathcal{E}^2} - 1\right), \qquad && R \to 0 \\ V_{\rm eff} &\sim \frac{4\pi^2\tau_2^2}{\mathcal{E}^2}R\,, && R \to \infty\,, \end{aligned} \tag{3.50}$$

Since $\frac{\mathcal{E}^2}{4\pi^2} > (B^2\tau_2^2 + p_u^2)$, we see that the effective potential approaches zero in the limit $R \to 0$ from below. In the same way we also see that the potential blows up for $R \to \infty$. It then follows that the behavior of the supertube in this background is the same as in the case of D2-brane background studied in the previous section. Therefore we skip the details of the discussions in this case.

3.3 NS5-brane background

As the next example we will consider supertube in the background of N NS5-branes. The metric, the dilaton and NS-NS field are

$$\begin{aligned} ds^2 &= dx_\mu dx^\mu + H_{NS}(R) dx^m dx^m \,, \\ e^{2(\Phi-\Phi_0)} &= H_{NS}(R) \,, \\ H_{mnp} &= -\epsilon^q_{mnp} \partial_q \Phi \,, \end{aligned} \tag{3.51}$$

where $H_{NS} = 1 + \frac{N}{R^2}$ is the harmonic function in the transverse directions of N NS5-branes. If we again restrict to the homogenous modes the hamiltonian density takes the form

$$\begin{aligned} \mathcal{H} &= \sqrt{p_R \frac{1}{H_{NS}} p_R + p_u g^{uv} p_v + \Pi^2 + b_\phi \frac{1}{R^2 H_{NS}} b_\phi + \frac{\tau_2^2}{H_{NS}} (R^2 H_{NS} + B^2)} \\ &= \frac{1}{\sqrt{N+R^2}} \sqrt{p_R^2 R^2 + p_u p^u + (N + R^2 + B^2)(\Pi^2 + \tau_2^2 R^2)} \,, \end{aligned} \tag{3.52}$$

where we have used

$$b_\phi = F_{\phi z} \pi^z = -B\Pi \tag{3.53}$$

and the fact that $\pi^z \equiv \Pi$ and p_u are conserved. We must also stress that we consider the situation when modes X^m that were defined in previous section, are not excited and equal to zero.

Now the equation of motions take the form

$$\begin{aligned} \dot{A}_z &= E = \frac{\sqrt{-g_{00}}}{\sqrt{\mathcal{K}}} (g_{zz} + g^{\phi\phi} B^2) \Pi^2 \,, \\ \dot{A}_\phi &= 0 \,, \end{aligned} \tag{3.54}$$

where the second equation above implies that $B = \partial_z A_\phi$ is conserved as well.

In order to study the dynamics of the radial mode we use as in the previous sections the fact that the energy is conserved. Firstly, if we write the hamiltonian density as $\mathcal{H} = \sqrt{p_R g^{RR} p_R + V}$ where

$$V = \frac{1}{N+R^2} \left(p_u p^u + (N + R^2 + B^2)(\Pi^2 + \tau_2^2 R^2) \right) . \tag{3.55}$$

Now we can express p_R from $\mathcal{E} = 2\pi\mathcal{H}$ as

$$p_R = \frac{\sqrt{g_{RR}}}{2\pi} \sqrt{\mathcal{E}^2 - 4\pi^2 V} \tag{3.56}$$

and hence the equation of motion for $\dot{R}$ is

$$\dot{R} = \frac{p_R g^{RR}}{\mathcal{H}} = \frac{1}{\mathcal{E}\sqrt{g_{RR}}}\sqrt{\mathcal{E}^2 - 4\pi^2 V} \Rightarrow \frac{2}{2}\dot{R}^2 + V_{\text{eff}} = 0\,, \tag{3.57}$$

where

$$\begin{aligned} V_{\text{eff}} &= \frac{1}{g_{RR}}\left(\frac{4\pi^2 V}{\mathcal{E}^2} - 1\right) \\ &= \frac{R^2}{N+R^2}\left(\frac{4\pi^2}{(N+R^2)\mathcal{E}^2}\left(p_u p^u + (N+R^2+B^2)(\Pi^2 + \tau_2^2 R^2)\right) - 1\right). \end{aligned} \tag{3.58}$$

For small and for large R this potential reduces to

$$\begin{aligned} V_{\text{eff}} &\sim \frac{R^2}{N}\left(\frac{4\pi^2}{\mathcal{E}^2 N}(p_u p^u + (N+B^2)\Pi^2) - 1\right), \qquad && R \to 0\,, \\ V_{\text{eff}} &\sim \frac{4\pi^2 R^2}{\mathcal{E}^2}\,, && R \to \infty\,. \end{aligned} \tag{3.59}$$

Again we see that the potential V_{eff} approaches zero for $R \to 0$ from below and blows up for $R \to \infty$. Consequently we obtain the same picture of the supertube dynamics as in the previous sections.

3.4 Macroscopic fundamental string background

In this section, we study the time dependent dynamics of the tube in the Macroscopic fundamental string background, where the metric, dilaton and the NS-NS charge of the F-background is given by

$$ds^2 = \frac{1}{H_f(r)}\left(-dt^2 + dz^2\right) + \delta_{mn}dx^m dx^n\,, \qquad B_{01} = H_f^{-1} - 1\,, \qquad e^{2\Phi} = H_f^{-1}\,, \tag{3.60}$$

where $H_f = 1 + \frac{N}{r^6}$ is the harmonic function in the transverse eight-space of the N F-strings. r denotes the spatial coordinate transverse to the macroscopic string.

Let us again consider the hamiltonian density for D2-brane

$$\begin{aligned} \mathcal{H} &= \sqrt{-g_{00}}\sqrt{\mathcal{K}} + \Pi^a B_{a0}\,, \\ \mathcal{K} &= (p_r + \pi^a B_{ar})g^{rs}(p_s + \pi^a B_{as}) + \pi^a \gamma_{ab}\pi^b + b_a g^{ab} b_b + e^{-2\Phi}\tau_2^2 \det \mathbf{A}_{ab}\,, \\ b_a &= \Pi_s \partial_a X^s + F_{ab}\pi^b\,. \end{aligned} \tag{3.61}$$

The hamiltonian density (3.61) is the starting point for our calculation where we consider D2-brane supertube stretched in z direction and that wind ϕ direction in the plane (x^8, x^9) in the space transverse to the fundamental string, where the modes that parameterize the embedding of the supertube, are time dependent only:

$$R = R(t)\,, \qquad X^m = X^m(t) \qquad m = 3, \ldots, 7\,. \tag{3.62}$$

Now thanks to the manifest rotation symmetry SO(6) of the subscpace $(x^3, \ldots, x^7)$ we can restrict ourselves to the motion of supertube in (x^3, x^4) plane where we introduce polar coordinates as

$$X^3 = \rho\cos\psi\,, X^4 = \rho\sin\psi\,. \tag{3.63}$$

Note also that the fact that we consider homogenous modes implies that A_z and Π are time independent.

Now the spatial matrix $\mathbf{A}_{ab}$ takes the form

$$\mathbf{A} = \begin{pmatrix} g_{zz} & B_{z\phi} \\ B_{\phi z} & g_{\phi\phi} \end{pmatrix} = \begin{pmatrix} H_f^{-1} & B \\ -B & R^2 \end{pmatrix} \tag{3.64}$$

hence its determinant is equal to

$$\det \mathbf{A}_{ab} = H_f^{-1} R^2 + B^2 \,, \qquad H_f = 1 + \frac{N}{(R^2+\rho^2)^3} \,. \tag{3.65}$$

We also consider the electric flux in the z direction only $\pi^z \equiv \Pi$ so that the term $\pi^a B_{aM} = \pi^z B_{zM} = 0$ using the fact that all fields do not depend on z. In the same way $b_\phi = F_{\phi z}\pi^z = B\Pi$. Now the term $\mathcal{K}$ takes the form

$$\mathcal{K} = p_R^2 + p_\rho^2 + \frac{p_\psi^2}{\rho^2} + \frac{\Pi^2}{H_f} + \frac{B^2\Pi^2}{R^2} + \tau_2^2(R^2 + H_f B^2) \,. \tag{3.66}$$

It is now easy to see that we can restrict ourselves to the motion when $p_\rho = p_\psi = \rho = 0$. Then we get

$$\mathcal{K} = \frac{(R^8 + B^2 N + B^2 R^6)}{R^6} \left(\frac{R^6 p_R^2}{R^8 + B^2 N + B^2 R^6} + \frac{\Pi^2 R^4 + B + R^6}{N + R^6} \right) . \tag{3.67}$$

Then finally the hamiltonian density takes the form

$$\mathcal{H} = \frac{1}{\sqrt{R^6+N}} \sqrt{(R^8 + B^2 N + B^2 R^6) \left(\frac{R^6 p_R^2}{R^8 + B^2 N + B^2 R^6} + \frac{\Pi^2 R^4 + B + R^6}{N + R^6} \right)} + \\ + \frac{\Pi N}{N + R^6} \,. \tag{3.68}$$

In order to simplify notation we will write the hamiltonian density for supertube in Fstring background in the form

$$\mathcal{H} = \sqrt{F(R) p_R^2 + G(R)} + \frac{\Pi N}{R^6 + N} \,, \tag{3.69}$$

where

$$F = \frac{R^6}{R^6 + N} \,, G(R) = \frac{(R^8 + R^6 B^2 + B^2 N)(R^6 + \Pi^2 R^4 + N)}{(R^6+N)^2} \,. \tag{3.70}$$

Then the differential equation for R is

$$\dot{R} = \frac{F p_R}{\sqrt{(\dots)}} = \frac{F p_R}{\left(\frac{\mathcal{E}}{2\pi} - \frac{\Pi N}{R^6 + N} \right)} \tag{3.71}$$

where we have expressed the square root using the conserved energy $\mathcal{E}$. If we also express p_R using $\mathcal{E}$ as

$$p_R^2 = \frac{1}{F} \left[\left(\frac{\mathcal{E}}{2\pi} - \frac{\Pi N F}{R^6} \right)^2 - G \right] \tag{3.72}$$

we get

$$\dot{R}^2 = \frac{F^2 p_R^2}{\left(\frac{\mathcal{E}}{2\pi} - \frac{\Pi N F}{R^6}\right)^2} = \frac{F}{\left(\frac{\mathcal{E}}{2\pi} - \frac{\Pi N F}{R^6}\right)^2}\left[\left(\frac{\mathcal{E}}{2\pi} - \frac{\Pi N F}{R^6}\right)^2 - G\right] \Rightarrow$$
$$\Rightarrow V_{\text{eff}} = \frac{F}{\left(\frac{\mathcal{E}}{2\pi} - \frac{\Pi N F}{R^6}\right)^2}\left[G - \left(\frac{\mathcal{E}}{2\pi} - \frac{\Pi N F}{R^6}\right)^2\right]. \tag{3.73}$$

Now we can perform the analysis exactly in the same way as in the previous section. Namely, the asymptotic behavior of V_{eff} is

$$V_{\text{eff}} \sim \frac{R^6}{N}\frac{1}{(\frac{\mathcal{E}}{2\pi} - \Pi)^2}\left[B^2 - \left(\frac{\mathcal{E}}{2\pi} - \Pi\right)^2\right], R \to 0\,,$$
$$V_{\text{eff}} \sim \frac{R^2 4\pi^2}{\mathcal{E}^2}, R \to \infty\,. \tag{3.74}$$

The potential approaches zero from below for $R \to 0$ on condition

$$\frac{\mathcal{E}}{2\pi} > B + \Pi \tag{3.75}$$

that is again obeyed. Consequently the analysis is the same as in the examples studied in the previous sections.

4. Summary and conclusion

In this paper, we have studied the time dependent dynamics of the supertube in various backgrounds, by using the Dirac-Born-Infeld effective field theory description in hamiltonian formalism. This provide yet another example of time dependent solutions of string theory in terms of some exotic bound of various D-branes with Born-Infeld electric and magnetic fields. The stability of these bound states in various curved backgrounds that are discussed in the literature recently, has been very suggestive in studying such dynamics and to view the trajectories of the supertube. We adopt the hamiltonian formalism for studying such dynamics of D-brane in general curved backgrounds in the presence of worldvolume gauge field. By using the crucial gauge fixing, we have studied the dynamics in Dp-brane, NS5-brane and fundamental string backgrounds and have analyzed the effective potential and the trajectory. It would be very interesting to study some other exotic bound states of branes (e.g. it has been argued in [30] that a string network can be blown up into a D3-brane, the so called supertube) in general curved background. There the analysis in terms on hamiltonian formalism seems much more complicated, but nevertheless a doable problem. It would also be very interesting to study the dynamics in Anti-de Sitter (AdS) spaces, and to study the effect in the dual conformal field theory by using the gauge-gravity duality. In the case of D-branes in AdS backgrounds, the tachyon like radion would be spatial dependent and hence studying the dynamics ought to shed light in studying the various properties of D-brane in curved backgrounds. We hope to return one or more of these issues in near future.

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